the posts, etc., outside the car seem passing by on account that we judge mostly by what is near our eyes. It is this that being stationary with respect to the earth, the ancients imagined the sun and stars to move around the earth instead of the opposite.

GEOMETRY.

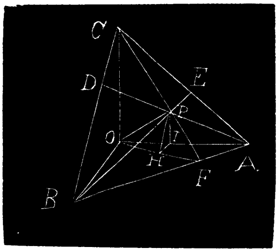
110. **Proposed by P. S. BERG, A. M., Principal of Schools, Larimore, N. D.**

If the three face angles of the vertical triedral angle of a tetraedron are right angles, and the lengths of the lateral edges are represented by a, b, and c, and of the altitude by p, then $1/p^2=1/a^2+1/b^2+1/c^2$. [Chauvenet's *Geometry*.]

I. **Solution by GEORGE R. DEAN, Professor of Mathematics, School of Mines and Metallurgy, University of Missouri, Rolla, Mo.; P. H. PHILBRICK, C. E., Lake Charles, La.; and CHAS. C. CROSS, Libertytown, Md.**

Let OC, OB, OA be the edges mutually at right angles; OP the perpendicular. Join AP and produce to meet CB at D; join BP and produce to meet AC at E; join CP and produce to meet AB at F.

Then $$\frac{\triangle APC}{\triangle ABC}+\frac{\triangle BPC}{\triangle ABC}+\frac{\triangle APB}{\triangle ABC}=1.$$



But $$\frac{\triangle PAC}{\triangle ABC}=\frac{PE}{BE},\quad \frac{\triangle BPC}{\triangle ABC}=\frac{DP}{AD},$$

$$\frac{\triangle BPA}{\triangle ABC}=\frac{PF}{CF}.$$

Therefore $$\frac{PE}{BE}+\frac{DP}{AD}+\frac{PF}{CF}=1.$$

$$BE\times BP=OB^2=b^2\,;\quad BE=\frac{b^2}{BP}\,;\quad \frac{PE}{BE}=\frac{PE\times BP}{b^2}=\frac{OP^2}{b^2}.$$

$$AD\times AP=OA^2=a^2\,;\; AD=\frac{a^2}{AP}\,;\quad \frac{DB}{AD}=\frac{AP\times DP}{a^2}=\frac{OP^2}{a^2}.$$

$$CF\times CP=OC^2=c^2\,;\; CF=\frac{c^2}{CP}\,;\quad \frac{PF}{CF}=\frac{PF\times PC}{c^2}=\frac{OP^2}{c^2}.$$

Hence $$\frac{OP^2}{a^2}+\frac{OP^2}{b^2}+\frac{OP^2}{c^2}=1,\text{ or }\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}.\qquad \text{Q. E. D.}$$

II. **Solution by P. S. BERG, B.Sc., Superintendent of Schools, Larimore, N. D.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; ELMER SCHUYLER, High Bridge, N. J.; W. H. WILSON, Professor of Mathematics, Geneva College, Beaver Falls, Pa.; and G. B. M. ZERR, A. M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

Let $OP=p$, $OC=a$, $OA=b$, $OB=c$, $OI=l$, $IH=m$, $PH=n$.

The triangles (right angled) OPA and OPI are similar, having $\angle POA$ in common.

$\therefore$ $p/b=l/p$, or $p^2/b^2=l^2/p^2$, or $p^4/b^2=l^2$.

Similarly, $p^4/a^2=n^2$, $p^4/c^2=m^2$.

$\therefore$ $p^4/a^2+p^4/b^2+p^4/c^2=n^2+l^2+m^2=p^2$.

$\therefore$ $1/a^2+1/b^2+1/c^2=1/p^2$.

Also solved by *J. SCHEFFER*, and *E. D. SCALES*.

111. **Proposed by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.**

Given that the area of a triangle is equal to half the product of two sides and the sine of the included angle, prove that $\sin(x+y)=\sin x\cos y+\cos x\sin y$.

I. **Solutfon by W. F. BRADBURY, A. M., Head Master, Cambridge Latin School, Cambridge, Mass., and B. F. YANNEY, A. M., Professsor of Mathematics, Mount Union College, Alliance, O.**

Given area $\triangle=\frac{1}{2}ac\sin B\ldots\ldots(1)$.

$\sin B=\sin[180^\circ-(A+C)]=\sin(A+C)$.

$\therefore$ Area $\triangle=\frac{1}{2}ac[\sin(A+C)]\ldots\ldots(2)$.

Draw BD perpendicular to AC.

Area $\triangle=\frac{1}{2}BD(AD+DC)$.

But $BD=c\sin A=a\sin C$, and $AD=c\cos A$, and $DC=a\cos C$.

$$\therefore \text{Area } \triangle=\frac{1}{2}\begin{matrix} c\sin A \\ \text{or} \\ a\sin C\end{matrix}(c\cos A+a\cos C)=\tfrac{1}{2}(ac\sin C\cos A+ac\sin A\cos C)\ldots(3).$$

Putting $(2)=(3)$, $\frac{1}{2}ac[\sin(A+C)]=\frac{1}{2}ac(\sin A\cos C+\cos A\sin C)$ or $\sin(A+C)$ $=\sin A\cos C+\cos A\sin C$.

II. **Solution by J. OWEN MAHONEY, B. E., Professor of Mathematics and Science, Carthage High School, Carthage, Tex.; J. W. YOUNG. Columbus, O.; J. SCHEFFER, A. M., Hagerstown, Md.; E. L. SHERWOOD, A. M., Professor of Mathematics, Whiteworth College, Miss.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; B. F. SINE, Principal of Normal School, Capon Bridge, W. Va.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; JOHN MACHNIE, A. M., Professor of Latin, University of North Dakota ; H. F. STRATTON, Student in Heidelberg University, Tiffin, O.; J. C. NAGLE, C. E., Professor of Civil Engineering and Physics in the Agricultural and Mechanical College of Texas, College Station, Tex.; CHARLES C. CROSS, Libertytown, Md.; and ELMER SCHUYLER, High Bridge, N. J.**

PROOF. Consider the triangle ACB.

The area of $ACB=\frac{1}{2}ac\sin B=\frac{1}{2}ac\sin(A+C)=\frac{1}{2}hb=\frac{1}{2}h(AD+DC)$, or

$$\sin(A+C)=\frac{h}{a}.\frac{AD}{c}+\frac{h}{c}.\frac{DC}{a}$$

$$=\sin C\cos A+\sin A\cos C.$$

$$\therefore\ \sin(x+y)=\sin x\cos y+\cos x\sin y.$$